Computational Method 2 Spring 2025

HW4

Q1 Numerical solution to the Schroedinger Equation.

Solve the Schroedinger Equation numerically and compute the energy spectrum (and the wavefunctions) of the Morse Potential:

$$V(x) = \frac{\lambda^2}{2m}\,(e^{-2x} - 2 * e^{-x}).$$

- a) (choose e.g. $m = 1$, $\lambda = 6$) Compute the ground state and the first 4 excited states. Check with the analytic result for the energy levels:

$$E(n) = -(\lambda - 1/2 - n)^2\,\frac{1}{2m}.$$

- b) Approximate the Morse potential with a harmonic potential. Check that their ground states agree. What about the 1st excited state?

Q2 Free Gas.

Review the computation of thermal pressure for free gas of bosons and fermions. Take $\mu_B = 0$.

a) Compute the thermal pressure for a gas of free pions (P_1). Plot P/T^4 as a function of T.

b) Compute the thermal pressure for a free gas of quarks (u and d) and gluons. (P_2). Work out their degeneracies. Plot P/T^4 on the same plot as part a).

c) Find the bag constant B such that

$$P_1(T_c) = P_2(T_c) - B$$

at $T_c = 0.16$ GeV.

d) Numerically construct a model equation of states such that:

P_model(T) = P_1(T) for T < Tc

P_model(T) = P_2(T) - B for T >= Tc

Show that P_model(T) is continuous at Tc, but the entropy density exhibits a jump. Plot the results.

e) Extract the latent heat L. The expected result is $L \approx 4 \times B$.